Nov. 13, 2002	MATH 425 Exam II	Fall Semester, 2002

Name:________________________ Score:__________________

1(15pts) (a) State the definition of $\lim_{x \to a^S} f(x) = L$.

(b) State the definition of uniform convergence for a sequence of functions f_n on $S \subset \mathbb{R}$.

(c) State the Cauchy criterion for uniform convergence of infinite series of functions on a subset: $\sum f_n(x), x \in S \subset \mathbb{R}$.

2(15pts) Prove that $\lim_{x \to a^S} f(x) = +\infty$ with $a \in \mathbb{R}$ if and only if $\forall N > 0,\ \exists \delta > 0$ such that $|x - a| < \delta,\ x \in S$ implies $f(x) > N$.

3(15pts) Consider $a_n = 3^{(-1)^n - n}$

(a) Find $\limsup a_n^{1/n},\ \liminf a_n^{1/n},\ \limsup \frac{a_{n+1}}{a_n},\ \liminf \frac{a_{n+1}}{a_n}$.

(b) Find the radius of convergence for $\sum 3^{(-1)^n - n} x^{2n+1}$.

(c) Determine the interval of convergence for $\sum 3^{(-1)^n - n} x^{2n+1}$.

4(15pts) Prove that a continuous function f on a bounded closed interval $[a, b]$ is uniformly continuous.

5(15pts) Let $f_n(x) = |x|^n$.

(a) Does f_n uniformly converge on $[-1, 1]$? Justify your answer.

(b) Does f_n uniformly converge on (a, b) with $-1 < a < b < 1$? Justify your answer.

6(15pts) Prove that the uniformly convergent limit f of a sequence of continuous functions $\{f_n\}$ on a subset $S \subset \mathbb{R}$ is a continuous function on S.

7(15pts) Do (a) or (b) but not both.

(a) State and PROVE the Intermediate Value Theorem.

(b) Show that if f is a continuous function on $[a, b]$, then there exists $x_*, x^* \in [a, b]$ such that $f(x_*) \le f(x) \le f(x^*)$ for all $x \in [a, b]$.

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